

## Infants' numerical abilities and models of early enumeration

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The early studies of infants' numerical abilities have shown that infants could discriminate numerosities and even form expectation about the outcome of a mathematical operation. In these studies, however, the number was confounded with continuous variables that co-vary with number, therefore subjecting this early research to a criticism. So, it was not clear whether infants possess genuine numerical abilities or not. In recent studies a control for continuous variables was introduced and it has been found that infants discriminate large numerosities, but not small ones. When small numerosities are confounded with other variables, infants discriminate them easily. It appears that infants represent small and large numerosities differently. Two systems of number have been proposed to account for the findings. One of the systems is responsible for approximate representation of large numerical magnitudes and the other for precise representation of small ones. Several mechanisms have been proposed to explain how these systems work. The systems are found across animals as well and could provide a foundation upon which more sophisticated, specifically human, mathematical abilities are built.

*Key words:* infants' numerical abilities, core systems of number, models of early enumeration

Adults are capable of performing arithmetical operations. They have gone through many years of formal schooling and learnt mathematical rules and concepts. The children, even before they start first grade, have mastered the basics: preschoolers know how to count and they know to do simple arithmetic operation such as addition and subtraction. In the last couple of decades researchers have tried to prove that the number domain develops before other cognitive capacities. A growing body of research shows that infants are granted with numerical abilities (Wynn 1998; Xu & Spelke, 2000; McCrink & Wynn, 2007). This paper is going to present that research. Studies that have found infants possess numerical abilities, such as enumeration or performing operations (addition and subtraction) on the enumerated numerosities will be considered. These studies will be critically examined. Also, models that account for infants' early numerical abilities will be reviewed.

### *Are infants capable of enumeration?*

The term *numerosity* is used in the literature to refer to a number of entities in a set, and *enumeration* is the process by which nonverbal animals and preverbal infants manage to determine the numerosity of a set (enumeration could be considered as a precursor of verbal counting). Infants are not capable of verbal counting, but they are very proficient in the domain of enumeration. Many researchers have reported that infants can discriminate numerosities (Starkey & Cooper, 1980; Wynn, 1996; Xu & Spelke, 2000). They are also capable of abstracting numerosities across modalities (Kobayashi, Hiraki, Mugitani & Hasegawa, 2004). Most of the studies are done with infants a couple of months old. The stimuli are varied through different modalities from study to study, ranging from dots on a screen, to a sequence of drum beats or jumps of a toy (Starkey, Spelke, & Gelman, 1990; Wynn, 1996; Xu & Spelke, 2000).

Starkey and Cooper (1980) showed that infants could discriminate small numerosities. They used the habituation-dishabituation paradigm. In this paradigm, a stimulus, for example, 2 dots on a screen, is shown a number of times. The dots vary in size and position. Infants at first spend some time looking at it, but after a while they habituate (get used to the stimulus) and decrease the looking time. After the habituation criteria have been satisfied, a test is introduced. Infants are shown a novel stimulus, for example 3 dots, and a familiar stimulus used during the habituation

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phase. It is assumed that infants will look longer at the novel stimulus, which would imply they recognize that they are different. In this paradigm infants rely on their enumeration abilities to decide whether there is a difference between the two stimuli.

In their study infants were found to discriminate small numerosities (1, 2 and 3) but not large ones (4 and 6). The mechanism underlying this ability was called *subitizing* (a very fast and accurate perception of small numerosities). The larger numbers were thought to lie outside of the subitizing range and therefore infants could not discriminate those two numerosities. The next step was taken by Starkey, Spelke and Gelman (1990) who also used the habituation-dishabituation paradigm. Instead of dots, they used more complex stimuli, such as photographs of different objects taken from a various distance. Again, they found that infants could discriminate numerosities. They took their research further by examining cross modal matching of numerosities using the *preferential looking* paradigm. In this paradigm infants are presented with a picture of two or three objects while playing two or three drum beats and their looking time is measured. When two drum beats were presented they tended to look more at the picture with two objects and when three drum beats were presented they tended to look at the picture of three objects. It appears that infants are able to detect numerical correspondence, but it may not be necessarily so. When the drum beats and the photographs are presented simultaneously infants could just match one drum beat with one object on one to one correspondence basis. In that case, they would not need to use enumeration abilities. However, if the drum beats are not presented simultaneously with the photographs, then infants' performance must rely on their enumeration abilities (Starkey et al., 1990). Indeed, what Starkey et al. (1990) found is that infants could match the numerical magnitude of two sets even when they were not simultaneously present (by first presenting photographs of either 2 or 3 objects, and then playing 2 and 3 drum beats while measuring the infants looking time at the sound source). For example, infants looked longer at the sound source from which 2 drum beats were coming when they first saw picture with two objects. This means that the infants were not only able to enumerate visually presented stimuli and enumerate audible stimuli, but they were also able to detect cross modal numerical correspondence between object and sounds. These results contribute to the growing body of evidence that infants are able to abstract numerosity.

Another evidence that infants abstract numerosity comes from Wynn (1996). Using the habituation-dishabituation paradigm she presented infants with rather unusual stimuli: a physical action (jumps of a toy). She had two groups of infants. One of the groups was habituated to 2 jumps and the other to 3 jumps. For example, the habituation began with a curtain raising up and revealing a puppet. The puppet made 2 jumps, pausing briefly between jumps, and then

stood still. At the end of a trial the curtain dropped for a short period of time and obscured the puppet, and immediately afterwards rose again to mark the beginning of the next trial. After the habituation criteria were satisfied the infants received six trials in which the puppet jumped 2 and 3 times alternately. As in the other studies, those who were habituated to 2 jumps dishabituated to 3 jumps and those who were habituated to 3 jumps dishabituated to 2 jumps. It appears that infants are able to regard numerosity as an abstract feature of a set of entities.

Another kind of evidence goes in favor of the idea that infants are able to abstract numerosity. It is not only individual entities that infants discriminate, but also a collection of entities. In a study conducted by Wynn, Bloom and Chiang (2002) it was found that infants could discriminate, for example, 4 groups of 2 objects and 2 groups of 4 objects. This means that the numerosity discrimination is not dependent on objects, but the characteristic of a number as a property of a collection of entities is abstracted, regardless of whether the entities are individual objects, collection of objects, or physical actions (Wynn, 1996).

In most of the studies mentioned so far, researchers have used only small numerosities. Starkey and Cooper (1980), however, used large numbers (4 and 6) and found that infants could not discriminate large numerosity, but in their study the numerical distance between the two large numbers was very small. That is why other researchers decided to test infants with large numbers with large numerical distance. Xu and Spelke (2000) presented infants with two conditions. In the first condition, 6 month old infants managed to discriminate 8 from 16 dots (1:2 ratio), but failed to discriminate 8 from 12 dots. It seemed as infants are able to discriminate large numerosities, provided the numerical distance between them is sufficient, that is, infants discriminate approximately. Others (Xu & Arriaga, 2007) have tested 10 month old infants with the 8 vs 12 (2:3 ratio) condition and found that these older infants were able to discriminate 8 from 12, but not 8 from 10 (4:5 ratio). Adults can discriminate ratios of 1:1.15 (Van Oeffelen & Vos, 1982, as cited in Lipton & Spelke, 2003). It appears that the ability to discriminate becomes better with age.

These infants' abilities are not characteristic of the humans only. They represent an evolutionary heritage because they are also found in the animal world. Animals have been found to discriminate approximately between numerosities (see Gallistel & Gelman, 1992, 2000; Dehaene, 1997 for a review) and the data gathered is similar to the data obtained with infants and humans.

#### *Do infants possess abilities that go beyond enumeration?*

Infants are not only capable of enumeration abilities, but they also have ordinal numerical knowledge. Infants of

11 months have been found to possess ordinal numerical knowledge, whereas 9 month old infants did not possess ordinal numerical knowledge, but had ordinal non-numerical knowledge (i.e. were able to discriminate the ordinal direction of sequence that varied in size, but not in number; Brannon, 2002). These findings suggest that the numerical ordinal competence is separate from the non numerical ordinal competence, and that the later develops first. It is also apparent that early primitive system exists not only for representing numerosities but for comparing them as well.

This simple enumeration and comparison process is not where infants' abilities terminate. Apparently, once infants have enumerated the entities they can also perform transformation operations. In a classical study, Wynn (1992) showed that infants can form expectation about the outcome of a mathematical operation, using the *violation of expectation* paradigm. In this paradigm infants are shown an object. Then, an obstacle is introduced in front of the object which hides the object from view. The infant could see the obstacle, but not what is behind it. Next, another object is put behind the obstacle while the infant watches. In half of the trials, one of the objects is secretly removed, without the infants being aware of it. In the other half, the original object and the added object remain. After the obstacle is removed, infants are faced with either possible ( $1+1=2$  objects) or impossible ( $1+1=1$  object) outcome. If infants have formed some expectation about the outcome ( $1+1=2$ ), then they should be surprised when they see the impossible outcome ( $1+1=1$ ) and thus spend longer time looking at that outcome than at the possible one. In another version of this paradigm, a subtraction operation can be substituted for addition operation. In the subtraction condition, more than one object is shown initially. Afterwards, one or more are removed and again infants are shown possible or impossible outcome. Wynn (1992) used this paradigm and reported that infants looked significantly longer at the impossible outcome, which would imply that infants have formed an expectation about the outcome of the mathematical operation. In this paradigm infants used their enumeration abilities, but they also manipulated the enumerated numerosities. In the study Wynn used small numbers (1, 2 and 3) and the infants formed exact expectations about the outcome. Earlier it was shown that infants represent numerical magnitudes approximately. It would be important to see whether these approximate representations extend to arithmetical operations as well, because if that is so, it would indicate a common mechanism in which both operations (enumeration and transformation such as addition or subtraction) work. For example, one can investigate whether infants would form expectations if larger numbers are used, and whether these expectations would exist for exact or approximate operations (for example  $6+10=14$  to be regarded as true and  $6+7=8$  as false). Indeed, a recent study has looked at this ability. McCrink and Wynn (2004), however, used exact large numbers ( $5+5=10$  or 5 and  $10-5=5$  or 10). They found that infants looked longer at the unexpected

outcome and concluded that infants possess procedures for numerical computations. Studies with approximate arithmetic have also been done, however not with infants. Such studies have been conducted with preschoolers and it has been shown that they are capable of approximate arithmetic (Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Gilmore & Spelke, 2008). Future studies will need to trace this ability back to infancy. If infants are found capable of approximate computation it would support a primitive computational system which serves as a basis for development of a more sophisticated exact computational system, a feature of humans only.

A recent study has shown that infants are capable of extracting ratios. McCrink and Wynn (2007) presented infants with different examples of the same ratio during the habituation phase. The stimuli were yellow Pac-Men and blue pellets that were presented in a single ratio during the habituation phase. In the test phase they presented either different example of the same ratio or different ratio. The infants looked significantly longer at the novel ratio, suggesting that they abstracted the ratio.

It appears that infants can not only enumerate sets, but also know that 1 is followed by 2 and 2 by 3. They can also "say" whether the outcome of an arithmetical operation (addition and subtraction only) is correct or not (by shorter and longer looking time, accordingly). Also, when presented with different examples of the same ratio, infants are able to extract the ratio. It almost appears as if infants are more prepared for mathematics than first graders are! This idea has encouraged many researchers to seek for alternative explanations for the infant's performance.

#### *Infants' abilities – a fatal flaw in design?*

Not all researchers agree that infants have numerical abilities. Mix, Huttenlocher and Levine (2002) reviewed the available literature regarding infants' enumeration abilities. They concluded that the researchers who reported that infants can discriminate numbers did not control for all the continuous variables that co-vary with number. For example, in their classical study Starkey and Cooper (1980) did not control for continuous variables such as contour length or total surface area. In that study, the stimuli used in both conditions (dots) were of the same size. This means that in the 3 dots condition infants did not only see more in terms of a number, but also in terms of total area (3 black dots on a white background fills up more space than 2 black dots on a white area). Because a greater number also meant greater surface area and greater contour length (circumference), in their study infants could have reacted to these other continuous variables that co-vary with number, and not to number. In other studies, where other type of stimuli was used, infants could react to other continuous variables. In the studies where sound stimuli were used, infants could use

non numeric cues such as tempo or duration. Same holds true for the jumps study: the total duration of the jumps was longer when there were three jumps than when there were two, therefore infants could react to duration and not to number. Mix et al. (2002) propose that infants react to difference in sets but they are guided by many cues, which under normal circumstances co-vary with one another. It is these combined cues that infants react to. This possibility re-opens the questions of whether infants possess genuine ability to abstract numerosities.

The studies using the violation of expectation paradigm have also been criticized. Feigenson, Carey and Spelke (2002) conducted a replication of Wynn (1992) experiment. In their study they controlled for volume, contour length and density and found that infants were reacting to novel continuous variables and not a number. In the pre-test phase they presented one large toy and in the post-test phase they presented either two smaller toys with the same surface area as the first (large) toy, or one smaller toy (same number, different area). According to Wynn (1992), the infants would look longer at the two smaller toys, but Feigenson's (Feigenson et al., 2002) findings show that infants looked longer at the toy with a different surface area but the same numerosity. Their results imply that infants reacted to other continuous variables, and not a number, that is they added other continuous variables and expected to see more of these, and not number.

Two more arguments could be given against the conclusion that infants are capable of enumeration and transformation of numerosities. The first one comes from another interpretation of the Wynn (1992) study. This interpretation is called the *object file model* and will be discussed in a later section. Briefly, it holds that infants just made same/different judgment without any reference to a number. Third interpretation is that infants just look longer at the more familiar and more complex stimuli (Cohen & Marks, 2002).

Finally, one should be very cautious when prescribing greater numerical abilities to infants than to children attending first grade. First of all, in all studies the main indicator for whatever is being measured is the looking time. But we know very little of what are the rules that guide infants' watching (Mix, 2002). When making claims such that infants are capable of arithmetical operations, we need much firmer proof than just measuring their looking time. For example, in the cross modal matching studies, it was taken that infants determine equivalence of the numerical magnitude of the sets because they looked longer at the sound source which emitted the same number of drum beats as the number of objects shown in a photograph. But if the goal of the research was to prove that infants discriminate 2 drum beats from 3 dots, how can we interpret the results? Do infants do not discriminate numerosities presented in different modalities or they do, but prefer to match (and look longer at the same numerosities) rather than to discriminate them (and look longer at different numerosity)?

Nevertheless, although most of the research regarding infants' numerical abilities has been subjected to a severe criticism, it is obvious from different paradigms that infants do possess genuine numerical abilities, though in rudimentary form.

In the next section studies that controlled for continuous variables will be presented. These studies showed that infants do discriminate numerosities and are able to perform arithmetical operations, therefore ruling out all other explanations of infant's performance in these tasks.

### *In defense of infants' numerical abilities*

Several arguments can be given against the claim that infants do not discriminate numerosities. Brannon and Cordes (2008) habituated infants to constant cumulative area while varying the numerosity (set size of one, two, or three elements) and tested the infants with a threefold and fourfold change in cumulative area and found that infants could not discriminate threefold change in area, but did discriminate fourfold changes. They repeated the experiments with larger numbers and found the same results. In contrast, infants easily discriminate twofold changes in a number when habituated to stimuli with constant number but varying area from slide to slide (Brannon, Abbott, & Lutz, 2004). Therefore, it would seem that infants discriminate changes in a number much more easily than they notice changes in cumulative area, which means that they discriminate a number and not continuous variables.

Another argument against the findings that infants have no numerical abilities comes from Hurewitz, Gelman and Schnitzel (2006) who have found that in adults the performance on tasks that require discrete numerical comparisons are affected by variation of stimuli along a continuous quantity (size). They argue that children may find it difficult to disregard the irrelevant dimension in experiments that systematically vary size, contour area or number. Therefore, they think that the studies which demonstrated that children prefer area over number do not reflect a lack of numerical competence but limits in inhibitory processing where infants are unable to disregard the area.

Xu (2003) conducted two experiments in which she studied infants' numerical discrimination while controlling for total area and contour length. In the first experiment she showed that infants could discriminate two large numerosities (4 and 8), but in the second she failed to show that infants discriminate small numerosities (2 and 4). This study is important because the same procedure and the same kind of stimuli were used in the two experiments and the same variables were controlled, therefore providing the first direct comparison between the ability to discriminate large and small numerosities. In this study only large numerosities (with sufficient numerical distance) could be discriminated when controlled for other continuous variables, but not



small numerosities. Other studies (Starkey & Cooper, 1980; Starkey, Spelke & Gelman, 1990) have shown that when continuous variables are confounded with (small) numbers infants can discriminate. The results point to the idea that two systems for numerical representation are present in infancy.

A finding by Brannon, Lutz and Cordes (2006) supports the idea of a common representational system for a number and continuous variables. In their study, 6 month old infants detected 1:2 ratio change in area of a single object (not cumulative area, which would require adding the areas of more than one object), but not 2:3 ratio change. This is interesting because infants at that age are able to detect twofold change in number and duration, suggesting that the three variables belong to a common representational system underlying number, area and time discrimination (Brannon et al., 2006)

In order to account for the available data that infants and adults can discriminate small numerosities when confounded with other continuous variables, but only approximately discriminate large numerosities it has been suggested that two core systems of numbers exist (Feigenson, Dehaene, & Spelke, 2004). One system is responsible for "keeping track of small number of individual objects and for representing information about their continuous quantitative properties" (Feigenson et al., 2004 p. 310). The other system is the system for approximate representation of numerical magnitudes, often referred to as *number sense*. The systems are thought to be common across animals, infants and adults. The approximate representation of a numerical magnitude is accompanied by the so-called distance and magnitude (or size) effect. The distance effect refers to the fact that discrimination between two numerosities becomes harder as the numerical separation between the two numerosities decreases (it is harder to discriminate 5 from 6 than 5 from 10). The size effect refers to the fact that for equal numerical distances discrimination decreases as the numbers become larger (it is harder to discriminate 9 from 10 than 2 from 3) (Dehaene, 1997).

Regarding infants abilities for performing operations on the numerosities, it was earlier stated that the object file model is an equally plausible explanation for their performance as the assumption of numerical abilities. However, the object file model has limitation in the number of items it can keep open which is about three (see the next section). Therefore, if larger numbers are used and infants still perform equally well, the object file can not be the correct explanation for their abilities to perform computations. McCrink and Wynn (2004) used larger numbers ( $5+5=5$  or 10) and found that infants did look longer at the unexpected outcome ( $5+5=5$ ), providing more convincing evidence that infants possess numerical competence.

Most of the studies suggest that we are born with an innate capacity (or capacity that develops very early in the ontogeny) to understand numerosities. The two core systems

of number provide the basic meaning of a number and are a necessary precondition for learning the associations between the perceived numerosity and the symbol representing it, such as a numerical word or Arabic digits (von Aster & Shalev, 2007). Young children's verbal counting builds on infants' enumeration abilities. As infants become toddlers and then preschoolers they acquire the exact meaning of each number word. At first they know that number words refer to numbers but do not know to which number they refer. It takes about one year since they have mastered the meaning of the word "one" until they learn the meaning of all number words within their counting range. Perhaps this slow learning process is a result of the approximate representation of number where close numerosities are often confused with one another. As young children grow older these two systems probably merge and humans are capable of representing exact large numerosities.

Next, three theoretical models of early enumeration are going to be considered. Each of these models explains how one or the other core system works.

#### *Theoretical models of early enumeration*

Several models have been proposed to account for infants' enumeration abilities. Here, three of them will be presented: the accumulator model, the neuronal network model and the object file model. The first two assume an analog representation of a number and they explain the approximate representation of a number. The object file model from the analog representation models differs in the nature it represents numerosity. Unlike the other two models, it does not represent number, but represents objects. This model accounts for the precise representation of small numerosities.

*The accumulator model* operates by emission of pulses at a constant rate. The model was first proposed by Meck and Church (1983) to account for rats' numerical competence. Later it was applied to infants' preverbal numerical competence. The model works as follows: to begin counting a switch is turned off to channel the impulses in the accumulator for a brief period. The switch is opened and closed for each entity to be counted. Since the impulses are emitted at a constant rate the amount of impulses that enter the accumulator for each item is equivalent. The numerosity of the items counted is represented by the fullness of the accumulator which consists of all the increments. After the counting has finished the total amount in the accumulator is proportional to the number of entities counted. This means that the relationships between the numerical quantities are exactly reproduced by the representations of the numerical quantities (Wynn, 1998). For example, five is three more than two, and the representation (the fullness of the accumulator) for five is three more increments than the representation for two. However, due to an inherent error in the enumeration process, such as the variability of the amount of time the switch stays closed or the rate at which the impulses

are emitted, the fullness of the accumulator after counting a given numerosity will vary from count to count and this variability will be normally distributed around a mean fullness. As the numerosity increases so will the variance of the distribution (Wynn 1998). That means that the numerosities are represented approximately. This explains why infants can reliably discriminate some ratios but not others. For example, 6-months-old infants cannot discriminate 8 from 12 (2:3 ratios) (Xu & Spelke, 2000). Presumably, their representations of these two numerosities are too noisy and cannot be discriminated from one another. A useful analogy when thinking about this is the level of water in a measuring cup. If the measuring cup is still, the water level can be read against the markings of the cup (let us say 150 ml). But if the cup is shaken the water level will fluctuate between 100 and 200 ml, that is, the representation is too noisy. Similarly happens with the representations of the numbers, yet, the larger the number the greater the fluctuation.

According to the accumulator model, addition could be achieved by pouring the content of two accumulators into a third empty one. Similarly, subtraction could be achieved by removing the content from one accumulator (Vilette, 2002). This explains how infants can form expectations about the outcome of an arithmetic operation.

The *neuronal network model* was proposed by Dehaene & Changeux (1993). If the accumulator model was mechanistic, the neuronal network model is biologically based. Its main component is the numerosity detection system which consists of a retina, a map of object locations and numerosity detectors. Objects of differing sizes and locations are represented on the *retina*. Because these objects (the input) differ in size, shape and other features, the input is normalized in order for each entity to be counted as one object regardless of size, shape etc. This normalization is accomplished by allocating approximately constant number of neurons to each object on a *map of object location*. Once normalization has taken place the total neuronal activity in the normalized location map is estimated. That estimate corresponds to the fullness of the accumulator in the accumulator model. The summation of the total neuronal activity is done by *numerosity detectors* in the parietal lobe. Each detector reacts to certain total activity it receives which falls within a range that varies from detector to detector. That means that some numerosity detectors are tuned to few specific numbers and will react only to them. Therefore each neuronal detector will react to a certain approximate number of objects (Dehaene, 1997). For example a neuron may react optimally to the number 8, less optimally to 7, 6, 9 and 10 and even less optimally to 5, 4, 11 and 12. Because some numerosities cause firing of the same neurons (although less optimally) they are indistinguishable from each other, hence infants cannot discriminate them.

The *object file model* suggests that infants demonstrate success in number tasks via nonnumeric methods. In order to represent the items that need to be enumerated infants

use mental tokens. This model holds that the properties of each perceived object such as color, size and location are encoded in a separate file. Each time a new object appears a new object-file is opened (Vilette, 2002). Two objects would be encoded by the representation of the form "Yi Yj" because there is no single symbol for two. The infants have no arithmetic procedures in their mind, but only the ability to manipulate mental representations of objects. For example, in Wynn's (1992) experiment, infants activated one token or object-file for the first object they saw. After the screen was raised and another object was added the infants activated another object-file and had two object-files open. When the screen was lowered infants were faced with one object and created object file representation for the unexpected outcome (activated one object-file). They compared the initial state with the present state on a one-to-one correspondence basis and noticed discrepancies (Feigenson et al., 2002). The object-file model is limited in the number of object-files that it can keep open, which is about 3 for children and 4 for adults. This explains why numbers up to 3 can be represented exactly, but afterwards the representation becomes fuzzier.

In this section three models that explain infants' enumeration abilities were considered. Some models describe the functioning of the first core system, and some the functioning of the second core system of numbers. One of the models is referred to as non-verbal counting because it works in a similar way to counting: moving one number up by adding one increment. This might suggest that children possess intuitive counting knowledge and that they know the underlying concept of counting before they learn how to count

## Conclusion

In this paper it was taken a view that infants possess genuine numerical abilities. In the last couple of decades researchers have provided convincing evidence that infants are capable of enumeration. Infants have been found to discriminate small numerosities exactly, when other co-varying variables are not controlled for and large numerosities approximately when other co-varying variables are controlled for. Because infants represent small and large numerosities differently, two systems for number have been proposed. The two systems share evolutionary heritage with the animal species.

Infants are also capable of performing arithmetical operations, such as addition and subtraction. Alternative explanations for their performance have been ruled out by a study which controls for continuous variables and which makes the object file explanation unlikely. Other studies have found infants to possess ordinal numerical knowledge and showed that they can abstract a ratio when shown a different example of the same ratio.

Some of the studies with infants have been subjected to a criticism because of a flaw in the experimental design. The

early studies did not control for continuous variables but the more recent ones did, and they show that infants do possess genuine numerical abilities. Perhaps, the biggest criticism comes from the chosen indicators: the looking time. It is argued that the rules by which infants look at, are still unknown. However, the fact remains that, when the same procedure is used, infants could discriminate large numbers but not small ones. Furthermore, the evidence from the habituation-dishabituation paradigm, preferential looking paradigm, and the violation of expectation paradigm, all point to the fact that infants possess numerical abilities.

Three models that account for infants' performance were presented. The object file model does not presume numerical abilities and it explains infants' performance on the same-different basis. When infants are tested in a habituation-dishabituation paradigm with small numbers it is likely that they utilize this mechanism. On the other hand, when they need to discriminate large numbers their performance is based on the accumulator model or the neuronal network model, which make very similar predictions.

Although most authors would agree that infants possess numerical abilities, they are far from the numerical abilities found in adults. Infant's early and rather primitive abilities are found in the animal kingdom as well, and they serve as a foundation on which all specifically human numerical abilities are later built.

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